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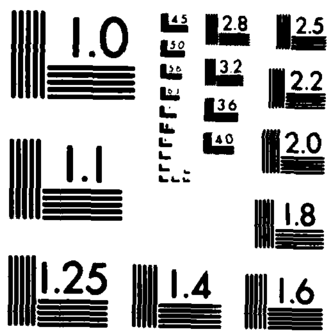

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARO 21096.3-MA	2. GOVT ACCESSION NO. N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) Poisson and Multinomial Shadowing Processes		5. TYPE OF REPORT & PERIOD COVERED 1 Oct 83 - 30 Sep 84 Final Report
		6. PERFORMING ORG. REPORT NUMBER
AUTHOR(s) Shelemyahu Zacks		8. CONTRACT OR GRANT NUMBER(s) DAAG29-83-K-0176
PERFORMING ORGANIZATION NAME AND ADDRESS State University of New York at Binghamton		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE December 1984
		13. NUMBER OF PAGES 7
MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Poisson Processes, Shadowing, Approximating, Probability, Target Detection, Visibility, Targets,

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During the period October 1, 1983 - September 30, 1984 the research concentrated on problems connected with shadowing processes in the plane. Three technical reports were issued. The titles ~~of these reports~~ are:

abstract continued

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20. ABSTRACT CONTINUED:

(1) The Distribution of A Weighted Visibility Measure On A Line Segment Under Shadows Cast By Random Disks Having A Bivariate Normal Scattering; ~~(~~January 15, 1984)

(2) The Visibility of Stationary and Moving Targets In The Plane Subject To A Poisson Field Of Shadowing Elements; and (February 15, 1984)

(3) Approximating The Probabilities of Detecting And Of Hitting Targets And The Probability Distribution Of The Number of Trials Along the Visible Portions Of Curves In the Plane Subject To A Poisson Shadowing Process. (April 16, 1984)

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The present report summarizes the main problems that have been studied and the main results obtained.

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ARO 21096.3-MA

POISSON AND MULTINOMIAL SHADOWING PROCESSES

by

S. Zacks

Center for Statistics, Quality Control and Design

State University of New York at Binghamton

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POISSON AND MULTINOMIAL SHADOWING PROCESSES

by

S. Zacks

Final Report

December 21, 1984

U.S. Army Research Office Contract DAAG29-83-K-0176

Professor S. Zacks, Principal Investigator

CENTER FOR STATISTICS, QUALITY CONTROL AND DESIGN
University Center of the State of New York
at Binghamton, New York

1. Forward.

During the period October 1, 1983 - September 30, 1984 the research under contract ARO DAAG29-K-83-0176 concentrated on problems connected with shadowing processes in the plane. We issued three Technical Reports (50 copies of each report were submitted to the ARO Office). The titles of these reports are:

(1) The Distribution of A Weighted Visibility Measure On A Line Segment Under Shadows Cast By Random Disks Having A Bivariate Normal Scattering. (January 15, 1984)

(2) The Visibility of Stationary and Moving Targets In The Plane Subject To A Poisson Field Of Shadowing Elements. (February 15, 1984)

(3) Approximating The Probabilities of Detecting And Of Hitting Targets And The Probability Distribution Of The Number of Trials Along the Visible Portions Of Curves In the Plane Subject To A Poisson Shadowing Process. (April 16, 1984)

In the present Final Report we summarize the main problems that have been studied and the main results obtained. This is done in Section 2. In Section 3 we report the publications connected with the project.

The algorithms for the computations of the distributions of measures of visibility were coded in FORTRAN and delivered to USA TRASANA in White Sands Missile Range, for implementation.

2. Statement of Problems Studied And Main Results.

2.1. Technical Report No. 1.

Consider the following problem. A laser beam is oriented from a source at a point (x_0,y_0) to a line segment, T, whose center is located at the point (x_t, y_t). The energy beam is maximal at its center and is tapering off fast as the distance from the center increases. A common model for the distribution of the intensity of the energy around the center of the beam is the spherically symmetric Gaussian distribution, with very small standard deviation. Certain portions of the beam may be obstructed by obstacles which are randomly dispersed in the field. These obstacles could consist of different types of objects which are in the field, such as trees, bushes, piles of dirt, etc. The ray does not penetrate through such objects. If such an obstructing object intersects any ray from (x_0, y_0) to T, we say that the object casts a shadow on T. There may be different, sometimes overlapping shadows, which are cast on T by different objects in the field. The problem is to study and characterize the statistical properties of an integrated measure of energy that can penetrate the field and hit the target. We assume that the intensity of a ray connecting (x_0, y_0) with a point (x,y) on T is proportional to the normal (Gaussian) probability density function (PDF). The intensity function, w(x), is normalized, so that its integral from $x_t-\Delta$ to $x_t+\Delta$ is equal to 1. An integrated measure, W, of the random amount of light (energy) that reaches T from the source W is a random measure having a distribution which depends on the characteristics of the random field of the shadowing objects. We assume that the number of disks, N, is fixed (finite). Furthermore, given any partition of the plane to (Borel) sets $B_1, \ldots, B_m$, the number of disks $J_1, \ldots, J_m$, whose centers belong to $B_1, \ldots, B_m$, respectively, have a multinomial distribution, with probability vector (π_1, ..., π_m) which depends on the specified sets and on the stochastic scattering mechanism. In addition,each disk has a radius which is a realization of a random variable with a specified distribution. Such a model is called a multinomial field of shadowing objects. More specifically we assume that the centers of disks have coordinates which are independent random vectors having a given bivariate normal distribution, and that the radii of disks are independent and identically distributed random variables, independent of the center locations. The motivation for studying such a model is due to a particular military application, in which the shadowing objects are artillery rounds. N rounds are scattered according to a bivariate normal distribution around an aim point (x^*,y^*). Each round when exploded creates a cloud of dust and debris of random size. A planar cut of such a cloud is modeled as a disk. The model of the present paper can be further generalized to cases of several clusters of N disks, each one characterized by a different bivariate normal distribution. The paper develops algorithms for the determination of visibility probabilities for points on T. These visibility probabilities are required for the computations of the moments of W. A beta-mixture approximation to the distribution of W is provided.

2.2 Technical Report No. 2.

To illustrate some of the problems that can be solved by the methodology developed in Technical Report No. 2, consider the following examples:

Example 1: Visibility of Stationary Targets

An observer is placed at a given location in a forest, in order to detect specified targets (vehicles, animals, etc.). Due to the random location of the trees it is important to determine the probabilities that individual targets are observed and the distribution of the number of targets observed. For this purpose one has to determine the probabilities that any specified r points out of n, $1 \le r \le n$, are simultaneously visible. The locations of the targets may be specified or random.

Example 2: Visibility of Moving Targets

A target is moving along a specified path $\mathcal{C}$. An observer is located at a point $\underset{\sim}{O}$. Trees or other obscuring elements are distributed between $\underset{\sim}{O}$ and $\mathcal{C}$. It is often required to determine the distribution function of the total visible portion of $\mathcal{C}$.

Technical Report No. 2 presents the basic methodology for determining simultaneous visibility probabilities of several stationary targets from one observation point, and also how to determine the moments of a measure of the portion of time a target can be seen while it is moving along a curve in the plane.

2.3 Technical Report No. 3.

A hunter is trying to detect and hit a deer in a forest. Suppose that a deer is moving along a path in the forest and the hunter is located among the trees at some distance from the path. The path is only partially visible to the hunter; the invisible (shadowed) portion of the path is obscured by the trees which are dispersed randomly between the hunter and the path. A deer can be detected by the hunter if at least a certain part of it is visible. This occurs only if at least one of the visible segments of the path is sufficiently long.

After detection of a deer, in a visible segment, the hunter starts shooting. The deer continues, however, to move along the path in the same pace. During each shooting trial the deer crosses a length of τ of the path. Thus the number of shooting trials in each visible segment depends on the length of the segment. The shooting trials stop when the deer is either hit or enters an invisible portion of the path. When the deer enters another visible segment, it has to be detected again. For simplicity we assume that the shooting trials are Bernoulli, with probability of failure q, $0 < q < 1$.

The problem of deer hunting can be treated as a two dimensional shadowing problem. The hunter is located at a point $\underset{\sim}{0}$ in the plane, the deer moves along a curve C in the plane, and the trunks of trees can be described as random disks dispersed between $\underset{\sim}{0}$ and C. The methodology developed in the present paper is also applicable to three dimensional versions of the above problem.

In the present study we develop approximations for (a) the probability of detection; (b) the probability distribution of the maximal number of shooting trials N; and (c) the probability of survival of the deer (bird). We also provide numerical examples to illustrate the goodness of these approximations.

3. Publications.

The Technical Reports listed in Section 1 were submitted for publication. T.R. No. 2 was accepted for publication in the Journal of Applied Probability, and is expected to be in print by March 1985. T.R.'s No. 1, 3 were submitted for publication in the Naval Research Logistics Quarterly.

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